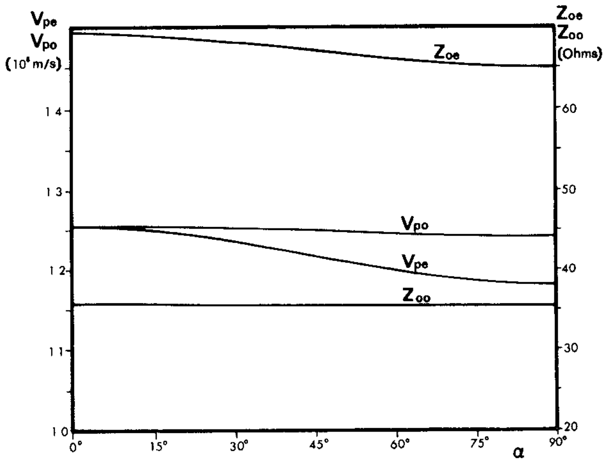

Fig. 5. 10-dB coupler, sapphire substrate. Coupled-line parameters versus rotation α. $\epsilon_{xx} = 11.6$, $\epsilon_{yy} = 9.4$ at $\alpha = 0°$, $B/H = 2.2$, $S/H = 0.225$, $w/H = 0.69$.

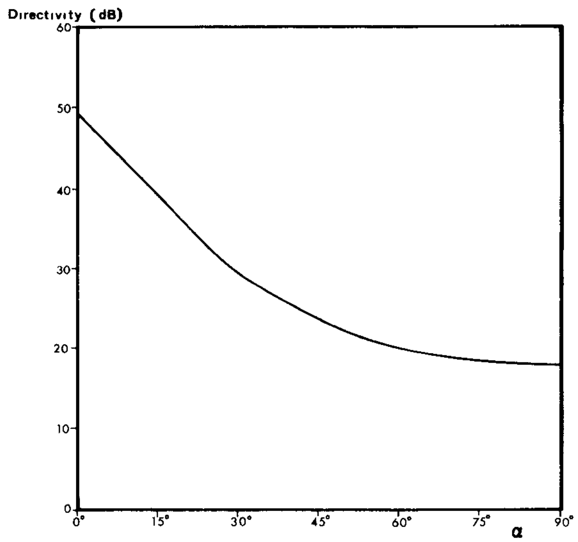

Fig. 6. 10-dB coupler, sapphire substrate. Directivity versus rotation α. $B/H = 2.2$, $S/H = 0.225$, $w/H = 0.69$, $\epsilon_{xx} = 11.6$, $\epsilon_{yy} = 9.4$ at $\alpha = 0°$).

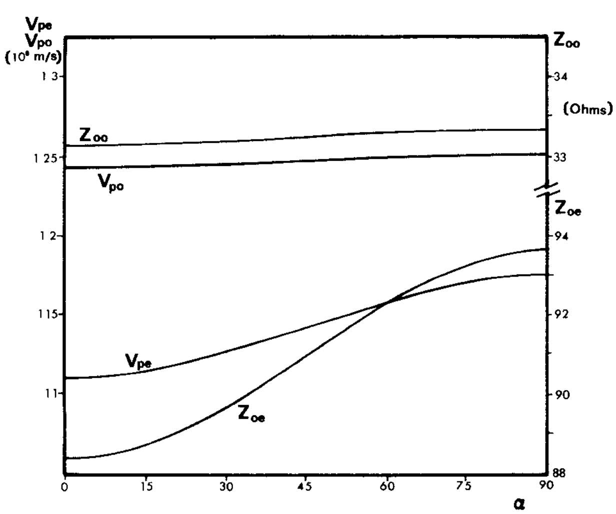

Fig. 7. 10-dB coupler, sapphire substrate. Coupled-line parameters versus crystal axis rotation α. $S/H = 0.1$, $B/H = 6.0$, $\epsilon_{xx} = 9.4$, $\epsilon_{yy} = 11.6$ at $\alpha = 0°$.

Comparable data for sapphire is presented in Figs. 5–7. No data for Epsilam as a function of crystal axis offset is presented, as it is an inherently aligned material.

IV. Conclusions

It has been shown that performance of coupled-line components, especially directional couplers, can be improved by using the anisotropy of certain substrate materials to equalize even- and odd-mode phase velocities. This can be accomplished by choosing a material with higher permittivity parallel to the ground plane than perpendicular, and lowering the top cover height. Materials with strong anisotropies are particularly suitable for this technique, and boron nitride in particular is a promising material. Sapphire is a less promising material, and couplers fabricated on sapphire substrates as conventionally fabricated may have worse directivity than those on isotropic substrates. For a sapphire substrate, high directivity is obtained when $B/H \simeq 1.9$; however, the performance is extremely sensitive to small tolerances of deviation from $B = 1.9\ H$.

Acknowledgment

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Propagation of Picosecond Pulses on Microwave Striplines

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Abstract —**Dispersion of picosecond pulses propagating on a microstrip line is calculated in the time domain. Numerical calculations are by the fast**

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Fourier transform, but a simple analytical model shows separation of high- and low-frequency components.

I. Introduction

Microstrip lines have been widely used as basic components of microwave integrated circuits. When the wavelength becomes comparable to the cross-sectional dimensions, dispersion becomes significant. Much work has been done to analyze this property in the frequency domain [1], [2] using Maxwell's equations. For computer analysis, a simple approximate formula to express the dispersion properties has been worked out by Yamashita *et al.* [3]. Kautz [4] has analyzed dispersion due to loss in superconducting striplines in the time domain. Relatively little attention has been paid to dispersion in microstrip lines fabricated on semi-conductors. These structures have recently been used in the field of picosecond optical pulse detection and picosecond electrical pulse generation [5], [6]. Pulses as short as 8 ps have been measured by auto-correlation from an amorphous silicon device [7]. These short pulses contain very high frequency components, and pulse distortions due to dispersion may be considerable. It is useful to see the effects on pulse shape of propagation over various distances in such lines.

II. Numerical Calculation

The microstrip geometry to be considered is shown in Fig. 1. To study the propagation of picosecond pulses, we consider a Gaussian pulse of the form

$$V(0,t) = V_0 e^{-(4\ln 2)t^2/\tau^2} \tag{1}$$

and its Fourier transform

$$V(0,\omega) = \sqrt{\frac{\pi}{4\ln 2}}\, V_0 \tau e^{-\omega^2\tau^2/4\ln 2} \tag{2}$$

where τ is the full width half maximum of the pulse. At a distance L, the pulse becomes

$$V(L,\omega) = e^{-j\beta(\omega)L} V(0,\omega) \tag{3}$$

where $\beta(\omega)$ is the propagation constant with attenuation being neglected. Inverting $V(L,\omega)$ gives the pulse $V(L,t)$ in the time domain.

A computer program using the fast Fourier transform (FFT) was written to calculate the dispersion effects. In the analysis, the total time interval used is 256-ps. By the properties of FFT, this means that the pulse repeats itself at 256-ps intervals. The maximum distance L_{max} in which this calculation is valid for a single pulse is limited by the fact that the low frequency components of a pulse catch up with the high frequency components of the previous pulse. In our case

$$L_{max} = 256 \times 10^{-12} \times \frac{3\times 10^{10}}{\sqrt{\epsilon(\infty)} - \sqrt{\epsilon(0)}} = 12.5 \text{ cm} \tag{4}$$

where $\epsilon(0)$ and $\epsilon(\infty)$ are the limiting values of the effective dielectric constants at low and high frequencies, respectively. The value of $\epsilon(\infty)$ is also the substrate relative permittivity ϵ_r.

The expression we used for ϵ as a function of frequency is that given by Yamashita *et al.* [3] and is graphed on Fig. 2 where the other parameters used are given as well.

Fig. 3 shows the results of the numerical calculations. Each frame is a computer simulated oscilloscope trace of a pulse at various distances along the microstrip line. For each trace, the time origin has been shifted by an amount $t_L = L/V_\phi(\omega = 0)$ where $V_\phi(\omega = 0)$ is the limiting phase velocity of the low frequency components. The evolution of the pulse shapes is very similar in all three cases with the serious distortion naturally occurring in shorter distances for the shorter pulses.

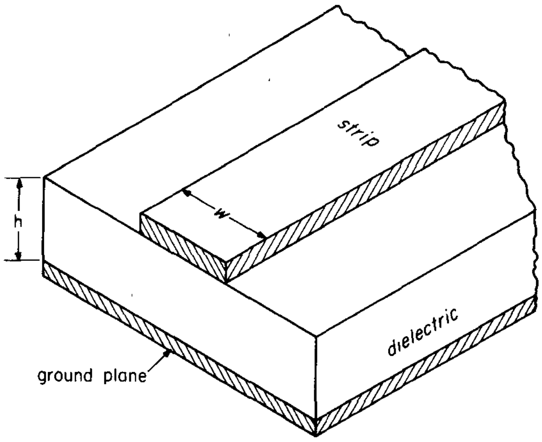

Fig. 1. Microwave stripline indicating major components and parameters.

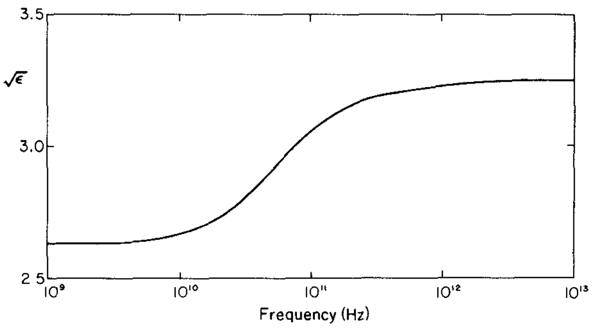

Fig. 2. Dependence of propagation constant on frequency of a microstrip line. Parameters used are $w = 320$ μm, $h = 400$ μm, $\epsilon(\infty) = 10.5$, $\epsilon(0) = 6.9$, and $z_0 = 50$ Ω.

III. Analytical Approximation

It is of course possible to represent the curve of Fig. 2 by piecewise analytic approximations that allow evaluation of $V(L,t)$, but this has no advantage over direct numerical calculation if the expressions become too complicated. One simple idealization that shows the clear separation of high and low frequency components is that illustrated in Fig. 4 in which low frequency components travel at one phase velocity and frequency components above some ω_0 at a lower phase velocity.

The value ω_0 is taken from the point of inflexion of $\phi(\omega)$ where $\phi(\omega) = \omega\beta/\beta_0$. Then, for an impulse input

$$V(L,\omega) = \frac{e^{-j\beta L}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = \frac{e^{-j\beta L}}{\sqrt{2\pi}}. \tag{5}$$

The inverse transform gives

$$V(L,t) = \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{-\omega_0} \frac{1}{\sqrt{2\pi}} e^{j\omega(t-\tau_2)} d\omega + \int_{-\omega_0}^{\omega_0} \frac{1}{\sqrt{2\pi}} e^{j\omega(t-\tau_1)} d\omega + \int_{\omega_0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{j(t-\tau_2)} d\omega \right\} \tag{6}$$

where

$$\tau_1 = \frac{\beta(0)L}{c\beta_0} \tag{7}$$

$$\tau_2 = \frac{\beta(\infty)L}{c\beta_0}. \tag{8}$$

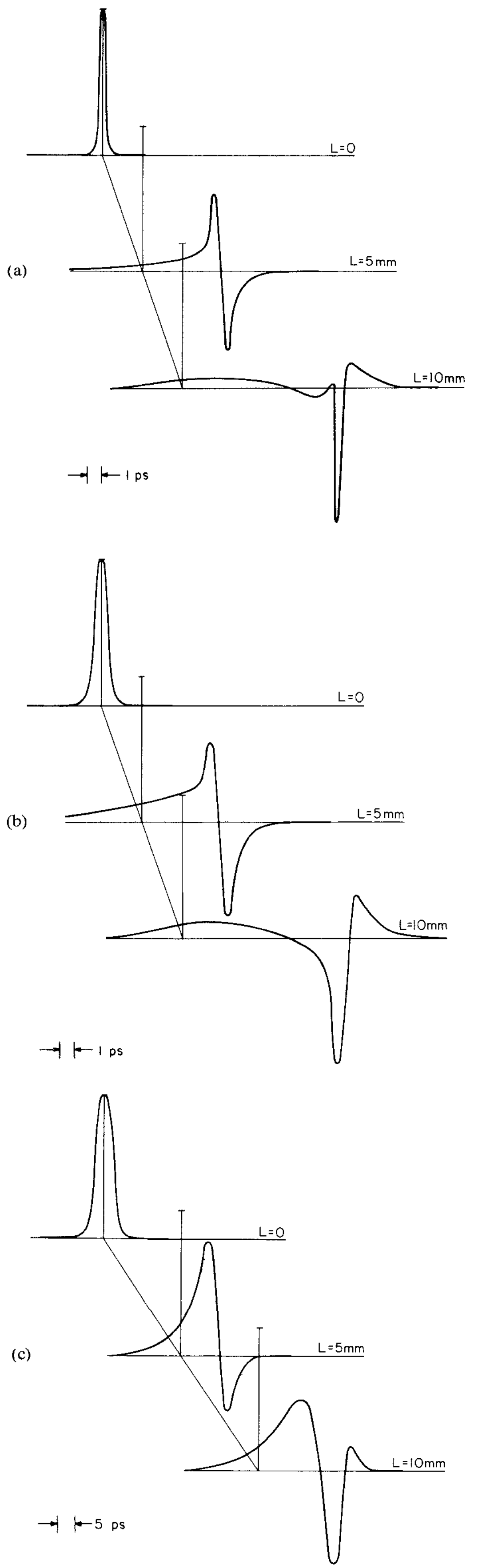

Fig 3 Propagation of Gaussian pulses for the example microstrip line. Initial pulse widths are (a) 0.5 ps, (b) 1 ps, and (c) 5 ps.

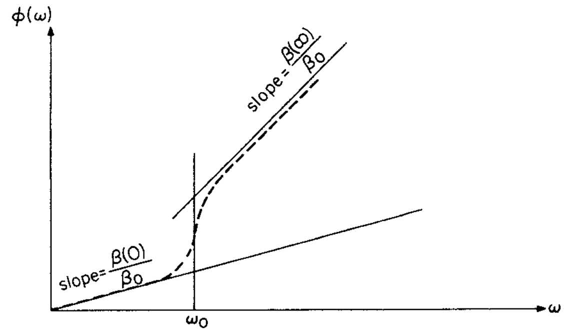

Fig. 4. Approximation used for phase $\phi(\omega)$.

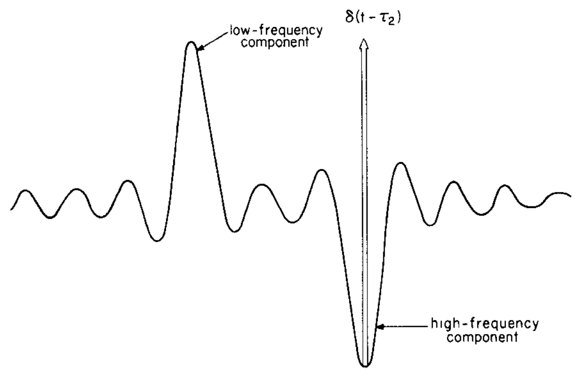

Fig. 5. Propagation of a δ-function along an idealized microstrip line of length $L = 1$ cm.

Rearranging and simplifying gives

$$V(L,t) = \delta(t-\tau_2) - \frac{\omega_0}{\pi}\frac{\sin\omega_0(t-\tau_2)}{\omega_0(t-\tau_2)} + \frac{\omega_0 \sin\omega_0(t-\tau_1)}{\pi\omega_0(t-\tau_1)}. \quad (9)$$

A typical output is shown in Fig. 5. It shows clearly the splitting of the high and low frequency components, similar to the numerical results. However, more pulse deformation is evident compared to the finite width pulses of Section II. Such simple analytical approximations thus have limited applicability when exact pulse shapes are desired, but they may be useful to give a qualitative feeling for the effects of dispersion.

IV. Analysis of Pulses Generated from Photoconductive Switches

Very short electrical pulses have been generated from photoconductive switches fabricated on amorphous silicon [7], GaAs, and InP in a microstrip-line structure. Because the *RC* time constant of the device is in the order of a few picoseconds, the risetime of the electrical pulse generated using a picosecond laser pulse can be less than 10 ps. The fall time, on the other hand, is determined by the decay of carriers in the semiconductor, which is usually long. The result is an asymmetric pulse. Fig. 6 shows numerically calculated changes of such a pulse as it propagates along the microstrip line. It is interesting to see that there is a sharpening effect in these pulses. Qualitatively, we can understand this by noting that the tail contains most of the low

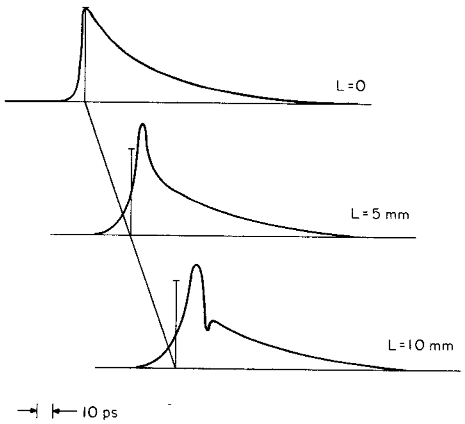

Fig. 6. Propagation of an asymmetric pulse for the example microstrip line.

frequency components which travel faster and catch up with the high frequency components in the rising edge.

V. Conclusion

Dispersion in a microstrip line has been analyzed numerically. The calculated output pulse shapes show how the pulses are distorted in the time domain. These give ideas in what to expect in picosecond electrical pulse generations. The approximate analytical formula shows the separation of the high and low frequency components and is useful in a qualitative sense. Application of this analysis to a pulse with practical parameters shows an interesting sharpening effect of the dispersion.

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The Equivalent Circuit of the Asymmetrical Series Gap in Microstrip and Suspended Substrate Lines

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Abstract —**The microwave properties of the series gap in microstrip and suspended substrate lines with unequal widths of the involved lines are described by means of suitable equivalent circuit data. These data have been computed using a rigorous three-dimensional spectral domain hybrid-mode approach developed by Jansen for the numerical characterization of the frequency-dependent scattering parameters of a wide class of strip and slot discontinuities. The results presented extend considerably the range of published gap data. In particular, they show that the stray-susceptances in the equivalent pi-network of the asymmetric series gap exhibit an inductive behavior for the case of tight coupling.**

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I. Introduction

Due to its importance for the end-to-end coupling of resonators in stripline and microstrip measurement configurations and filter structures, the strip series gap and its equivalent circuit have been treated repeatedly in the microwave literature of the past years (see, for example, references [1]–[11]). With the exception of a few selected results recently presented by the authors of this paper [12], the gap data available, however, have been derived by quasi-static computational methods or by measurements and are all restricted to the symmetric gap, i.e., the gap between the ends of strips of equal width [13]. For this type of symmetric strip discontinuity, it is commonly accepted that it can be modeled by a purely capacitive equivalent pi-circuit, independent of the degree of coupling, at least as long as the gap dimensions are small compared to wavelength. Physically, this means that the effect of the stray field at the ends of the coupled strips is dominant compared to the inductive current density disturbance created there, if strips of equal width are considered.

The paper presented here directs attention to the asymmetric series gap discontinuity which is shown to behave differently. The equivalent circuit data given in this contribution have been computed employing a rigorous three-dimensional spectral domain approach developed by Jansen [14] and recently proposed by Jansen and Koster as a unified CAD basis for the frequency-dependent characterization of MIC components [12]. The numerical approach used reduces the solution of the gap hybrid-mode problem to the satisfaction of electromagnetic boundary conditions in a small region around the gap. All other boundary conditions—for example, those on the exciting microstrip lines—can be satisfied in advance in preliminary phases of the computation, as is described in detail in [14]. The two-dimensional integral operator equation representing the boundary value problem is solved by Galerkin's method [12],[14].

It should be noted explicitly that the results achieved by the before-mentioned numerical approach are in terms of frequency-dependent scattering parameters and do not rely on the assumption of a special kind of equivalent circuit. Nevertheless, although the information inherent in scattering parameters satisfies the requirements of circuit design completely, there is a definite need for equivalent circuit data since in design considerations the use of these is much more convenient to microwave engineers. Moreover, suitably chosen equivalent circuits have the advantage that they are tightly related to physical interpretation and, therefore, numerically computed scattering parameters have been transformed into equivalent pi-circuit elements in this paper. The reference impedances used for the transformation, i.e., the characteristic impedances of the involved strip transmission lines, are also given for reasons of clarity and uniqueness. For the limiting case of equal widths, the computer program used here has been tested successfully by comparison with measurements and the results provided by other authors (see [12]).